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DYNAMIC STABILITY OF A HELICOPTER WITH HINGED ROTOR BLADES

By K. Hohenemser

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DYNAMIC STABILITY OF A HELICOPTER WITH HINGED ROTOR BLADES*

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SUMMARY

The present report is a study of the dynamic stability of a helicopter with hinged rotor blades under hovering conditions. While in this case perfect stability can in general not be obtained it is, however, possible by means of certain design features to prolong the period of the spontaneous oscillations of the helicopter and reduce their amplification, and so approximately assure neutral equilibrium. In contrast to rotors with blades mounted rigidly to the rotor shaft the gyroscopic effects of two rotors become additive, even if they rotate in opposite direction, and produce during a rotation of the helicopter a linear damping which is of the greatest importance for the stability and control characteristics of the helicopter. The possibility of controlled stability of a helicopter fitted with hinged blades is proved by the successful flights of various helicopters, particularly of the universally known Focke helicopter, FW61.

INTRODUCTION

For fixed-wing aircraft the solution of the problem of dynamic stability has been known for some time. For helicopters with blades fitted rigidly to the rotor shaft the case of hovering has been covered by H. G. Küssner. (reference 1). Having recourse to the test data of Flachsbart and Kröber (reference 2) on propellers in yaw he proved that dynamic instability is unavoidable on a helicopter without tail and with rigidly attached blades. Dynamic stability can be achieved by placing a suitably dimensioned empennage in the slipstream, but with counter-rotating rotors, for instance, it is still necessary to

*Über die dynamische Stabilität des Hubschraubers mit angelenkten Flügeln. Ingenieur-Archiv, December 1938, pp. 419-428.

have available a certain residual spiral of the rotors. In the following, the corresponding case of helicopter with blades hinged to the rotor shaft is treated. There is no tail, since on such a helicopter the blade loading is usually so low that a slipstream already effective with moderate tail surfaces does not result.

Air Loads

The helicopter has two coaxial, oppositely rotating, identical rotors placed so close together that the effect of blade clearance can be disregarded in the prediction of the air loads. Owing to the counter-rotating arrangement a movement of the helicopter in the longitudinal direction produces no air loads transverse to the direction of motion, so that longitudinal and lateral stability can be studied separately as on the fixed wing aircraft. Restricting the study to small motions, the square of the speed relative to the speed itself can be ignored. Since the thrust change on a blade in yaw is proportional to the square of the flow velocity (reference 3) the thrust remains unchanged in first approximation, hence no vertical motions occur. Thus the first longitudinal motion in question consists merely of a horizontal motion and a rotation about the transverse axis.

To set up the exact motion equations for the helicopter with hinged blades would result in a very complicated system, as each universally hinged joint even with utter disregard to its flexibility in bending involves two additional degrees of freedom. The following procedure avoids these difficulties: the air forces and mass forces on the blades are determined on the assumption that the rotor-tip plane preserves its position relative to the body during the motions of the aircraft. The actually occurring and computed changes of inclination of the rotor-tip plane relative to the body axis thus shall be small in ratio to the changes in inclination of the aircraft. Following this the inclination of the rotor-tip plane relative to the fuselage axis is defined on the assumption of uniform distribution of air forces and mass forces over the blades. That is, the aircraft motions shall be so slow as to approximately present a succession of steady stages for the helicopter blades. This premise prevents an increase in the number of degrees of freedom through the hinged blades, inasmuch as now the forces on the rotors are definitely dependent on the state of motion of the aircraft. Illustrative examples cited elsewhere in the report actually prove the assumptions approximately correct.

The air loads are computed by means of the simplified Glauert equations which are applicable to propellers with rectangular untwisted blades in yaw, when the direction of flow forms a small angle with the plane of the swept disk and when the rolling and longitudinal moments of the blades are equalized by a periodic blade feathering. For trapezoidal blades it is practical to assume a substitute rotor with rectangular blades whose chord is equal to the chord of the trapezoidal blades at 0.7 radius. In the same way it is proper for twisted blades to assume a substitute rotor with untwisted blades whose angle of attack is equal to the angle of attack of the actual blades at 0.7 radius. The most accurate theoretical values at present for the air loads on a rotor with hinged blades in yaw are obtainable with the formulas worked out by G. Sissingh (reference 4) for trapezoidal and for twisted blades. But, in a study of the dynamic stability linearized equations must be employed for the air loads. The illustrative examples computed with the exact formulas confirm, however, the complete practicability of the approximations employed here.

The rolling and longitudinal moment balance of a blade moving parallel to the plane of the swept disk requires a periodic change in blade angle of attack, where the angle of attack ϑ of the advancing blade decreases, that of the receding blade increases conformably to

$$\vartheta = \vartheta_0 - \vartheta_1 \sin \psi \qquad (1)$$

Here ϑ_0 is the mean angle of attack referred to zero lift curve, with the azimuth angle ψ being measured in peripheral direction from the rear position of the blade. The amplitude ϑ_1 of the periodic proportion of the blade-setting angle in relation to the coefficient of advance λ (ratio of forward speed to tip speed) follows approximately at

$$\vartheta_1 = 2\, \vartheta_0\, \lambda \qquad (2)$$

If the rotor blades are hinged to the rotor shaft so that the blades do not change angle of attack during flapping motions a backward inclination of the plane of the swept disk for an angle ϑ_1 results. For in that case the condition (1) relative to the new plane of the swept disk is satisfied (reference 5). In reality the hinged blades do not remain in one plane during rotation, but approximately describe a cone. Then the rolling and longitudinal moment balance requires other than the backward a lateral inclination of the rotor-tip plane toward the right or left, depending on the direction of rotation. But having assumed a pair of identical counterrotating rotors, the effects of these lateral inclinations cancel, hence may be disregarded in the following.

If proper design measures are taken so that the angle of attack decreases as a rotor rises and increases as it sinks, as suggested by Bréguet (German Patent No. 567,584, class 62b, 1933) the backward inclination of the rotor-tip plane is reduced. So, in order to include this case also, we assume a backward inclination of $\varphi \vartheta_1$, φ being a factor involved through the rotor design.

Since ϑ_1 is a small angle, the force perpendicular to the rotor shaft can be built up from a share $S \varphi \vartheta_1$ of the backward inclination of the plane of the rotor disk and a share $S\, k_{sn}/k_s$ representing the thrust component in the same plane. Here S is the total thrust of both rotors, k_s the thrust coefficient ($k_s = S/F\frac{\rho}{2}u^2$, where F is swept-disk area, ρ, air density, and u, tip speed), k_{sn} is coefficient of the thrust component in the plane of the rotor disk, computable from $k_{sn}/2\,\sigma = \lambda\, c_w/4$, where σ is solidity, i.e., ratio of blade area to swept-disk area, and c_w, the mean-blade drag coefficient. The value $k_s/2\,\sigma$ varies within comparatively narrow limits for practically designed helicopters. We assume $k_s/2\,\sigma = 0.09$, which, with $c_w = 0.016$, gives

$$\frac{k_{sn}}{k_s} = 0.044\ \lambda \qquad (3)$$

The force perpendicular to the rotor shaft with consideration to equation (2) then amounts to

$$S_n = S(\varphi\, \vartheta_1 + 0.044\,\lambda) = S\,\lambda\,(2\,\vartheta_0\,\varphi + 0.044) \qquad (4)$$

The hinges joining the blades to the rotor shaft are usually placed a certain distance a away from the shaft center. Owing to the backward inclination $\varphi \vartheta_1$ of rotor-tip plane toward the plane perpendicular to the shaft, a bending moment of magnitude $P_F \varphi \vartheta_1 a \cos \psi$, per blade of the corotating reference system results (P_F = centrifugal force of a blade). The longitudinal moment per blade on the aircraft is $P_F \varphi \vartheta_1 a \cos^2 \psi$. The proportions of this moment oscillating at twice the speed of rotation cancel, if each rotor has at least three blades, if with two blades a counteroscillating aircraft motion, not of interest here, is produced. The stationary proportions of the longitudinal moments of all blades are additive, ultimately yielding from $\varphi \vartheta_1$ with respect to the transverse axis through the centroid the tail-heavy moment

$$M_1 = \varphi \vartheta_1 \left(Ss + P_F \frac{a}{2} z\right) = 2 \vartheta_0 \lambda \varphi \left(Ss + P_F \frac{a}{2} z\right) \qquad (5)$$

Here s is distance of aircraft c.g. from the plane of the rotor disk and z, the total number of blades of both rotors.

With allowance for equation (3) the moment of the thrust component in the plane of the rotor disk with respect to the transverse axis through the c.g. is

$$M_2 = S s \frac{k_{sn}}{k_s} = 0.044 \; S s \lambda \qquad (6)$$

A third proportion of the longitudinal moment originates in the moments about the longitudinal axes of the blades. The moment coefficient c_m, referred to the aerodynamic center of the blade is independent of the blade angle of attack, whence the lift is assumed to apply at the aerodynamic center (about quarter chord). The counterforce for hinged blades is applied in the centroidal axis of the blade. Then the share of the moment dependent on the blade angle of attack is equal to the product of blade thrust and distance of the centroidal axis from the aerodynamic center of the profile. In the Fourier analysis of the blade thrust (blades being hinged) the term varying harmonically with the rotation is almost zero. But this very term is the only one in the development which, referred to the aircraft, has a stationary proportion. In consequence, the effect of the backward position of the center of gravity of the blade on the longitudinal moment of the airplane can be ignored. The share of the blade torque not dependent on the angle of attack is for rotating system

$$\int_0^R \frac{\rho}{2} t^2 c_m u^2 \left(\lambda \sin \psi + \frac{x}{R}\right)^2 dx$$

where R is rotor radius, t, blade chord, and x, distance of a point from the rotor center. The longitudinal moment of the airplane is obtained herefrom by multiplication with $\sin \psi$. The stationary quotas of all blades are additive again, the oscillating shares are disregarded, thus ultimately affording the longitudinal moment

$$M_3 = -\int_0^R \frac{z}{4} \rho\, t^2 c_m u^2 \lambda \times 2 \frac{x}{R} dx = -\frac{z}{4} \rho\, t^2 c_m R u^2 \lambda \tag{7}$$

For positive c_m, i.e., for a lift profile the blade moment and hence the longitudinal moment M_3 itself is nose-heavy, hence the minus sign.

The air loads on fuselage and other parts of the airplane, being proportional to the square of the speed, disappear for the presumedly small motions in relation to the forces and moments (equations (4) to (7)), which are proportional to the speed itself.

There remains the effect of the rotatory motion of the airplane about the transverse axis on the air loads. The variable angle of attack of the plane of the rotor disk relative to the wind direction does not change the result achieved so far, provided the motions are kept small, and hence small angles of attack of the plane of the rotor. But the rotation speed of the airplane about the transverse axis in the sense of a nose-heavy moment causes a lift increase in the advancing blade and a lift decrease on the returning blade. The result is an equalizing flapping motion corresponding to a right or left lateral inclination of the plane of the rotor tip, depending on the direction of rotation. The effects of these lateral inclinations cancel on account of the counter rotation of the two rotors. As a result of this the rotatory motion about the transverse axis produces no lateral air loads on the helicopter.

The Equations of Motion for the Longitudinal Motion

Let v represent the speed of the rotor center and α, the angle of rotation of the motion about the transverse axis. From the equilibrium of the forces perpendicular to the rotor shaft and the equilibrium of the longitudinal moments about the center of gravity (signs as in fig. 1) follow the equations

$$(\dot{v} - s\,\ddot{\alpha})\frac{G}{g} + \alpha\, S_{n\,\alpha} + \dot{\alpha}\, S_{n\,\dot{\alpha}} + v\, S_{n\,v} = 0 \tag{8}$$

$$\ddot{\alpha}\, J + \alpha\, M_{\alpha} + \dot{\alpha}\, M_{\dot{\alpha}} + v\, M_{v} = 0 \tag{9}$$

where $\dot{v}$ is horizontal acceleration, $\dot{\alpha}$, angular velocity, $\ddot{\alpha}$, angular acceleration, G, gross weight, g, acceleration of gravity, J, moment of inertia of airplane about the transverse axis, M, longitudinal moment and S_n, the force normal to the rotor shaft. For abbreviation, we write

$$\frac{\partial\, S_n}{\partial\, \alpha} = S_{n\,\alpha}$$

The other partial derivatives of S_n with respect to $\dot{\alpha}$ and v and of M with respect to α, $\dot{\alpha}$, and v are indicated by signs accordingly. With $S = G$ and $\lambda = v/u$, equation (4) gives

$$S_{n\,v} = \frac{G}{u}\,(2\,\vartheta_0\,\varpi + 0.044) \tag{10}$$

while equations (5) to (7) afford

$$M_v = \frac{2\,\vartheta_0\varpi}{u}\left(G\,s + P_F\,\frac{a}{2}\,z\right) + 0.044\,\frac{G}{u}\,s - \frac{z}{4}\,\rho\,t^2\,c_n\,R\,u \tag{11}$$

Now, outside of the air loads, the force of gravity and the mass forces of the rotors must be considered. The gravity component is $G\,\alpha$ and its longitudinal moment is zero. Hence

$$S_{n\,\alpha} = -\,G \tag{12}$$

$$M_{\alpha} = 0 \tag{13}$$

The rotation about the transverse axis is accompanied by gyroscopic moments which per blade referred to blade root have the magnitude $2\,J_F\,\dot{\alpha}\,\omega\,\sin\Psi$. ($\omega$ is rotation speed, J_F, inertia moment of a blade referred to blade root). In a rotation in the nose-heavy sense the gyroscopic moments raise the advancing blade regardless of the direction of rotor rotation. The rotor-tip plane is subject to a backward inclination the amount of which can be determined after equating the lift moment due to angle of attack change $\vartheta_1\,\sin\Psi$ to the gyroscopic moment

$$2J_F\dot{\alpha}\omega\sin\Psi = \int_0^R c_a'\vartheta_1\sin\Psi u^2\frac{\rho}{2}\left(\frac{x}{R}\right)^2 x t\,dx = c_a'\vartheta_1 u^2\frac{\rho}{2}\frac{R^2}{4}t\,\sin\Psi \tag{14}$$

c_a' is the lift gradient. The integration is effected for rectangular blade form, whence equations (4), (5), and (14) give

$$S_{n\dot{\alpha}} = S_{n\vartheta_1}\frac{\vartheta_1}{\dot{\alpha}} = \varphi\,G\,\frac{16\,J_F}{c_a'\,\omega\rho\,R^4\,t} \tag{15}$$

$$M_{\dot{\alpha}} = M_{\vartheta_1}\frac{\vartheta_1}{\dot{\alpha}} = \varphi\left(G\,s + P_F\,\frac{a}{2}\,z\right)\frac{16\,J_F}{c_a'\omega\,\rho\,R^4 t} \tag{16}$$

The gyroscopic forces produce a rotation damping moment on the rotor with hinged blades. If the rotors rotate in the opposite direction the gyroscopic effects are additive rather than neutralizing. This fact is of utmost importance for the flight characteristics of a rotating-wing aircraft with hinged blades.

The rotatory acceleration $\ddot{\alpha}$ about the transverse axis creates inertia moments which, for each blade referred to blade root, amount to $J_F\,\ddot{\alpha}\,\cos\Psi$. This moment induces right or left lateral inclinations of the plane of the rotor tip, depending on the direction of rotation. The effects of these inclinations cancel out because of the opposite rotation of the rotors. Thus the moment of inertia J of the aircraft about the transverse axis is taken without the blades.

Then the solution of equations (8) and (9) by means of

$$v = v_0 e^{\nu \tau} \quad \text{and} \quad \alpha = \alpha_0 e^{\nu \tau}$$

(τ denoting the time) gives two homogeneous linear equations for v_0 and α_0 which depend for their existence on the disappearance of the determinant of the coefficients. Then, allowance for equations (12) and (13) gives for ν:

$$\frac{G}{g} J \nu^3 + \left(M_{\dot\alpha} \frac{G}{g} + M_v s \frac{G}{g} + S_{nv} J\right) \nu^2 + \left(S_{nv} M_{\dot\alpha} - M_v S_{n\dot\alpha}\right) \nu + G M_v = 0 \qquad (17)$$

The condition for the appearance of decaying motions or oscillations only is a positive value of Routh's discriminant:

$$\left(M_{\dot\alpha} \frac{G}{g} + M_v s \frac{G}{g} + S_{nv} J\right) \left(S_{nv} M_{\dot\alpha} - M_v S_{n\dot\alpha}\right) - \frac{G}{g} J G M_v > 0 \qquad (18)$$

In addition, all coefficients in equation (17) must be positive. The factor ν in equation (17) is, with allowance for equations (10), (11), (15), and (16):

$$\frac{0.35 J_F G \varpi P_F a z}{c_a' \omega^2 \rho R^5 t} \left(1 - \frac{11.4 \rho t^2 c_n R u^2}{P_F a}\right) \qquad (19)$$

Up to M_v all other factors are also positive according to their physical significance, whence as sole further condition for the stability according to equations (11) and (19) the existence of the inequality

$$\frac{2 \vartheta_0 \varphi}{u} \left(G s + P_F \frac{a}{2} z\right) + 0.044 \frac{G}{u} s - \frac{z}{4} \rho t^2 c_n R u > 0 \qquad (20)$$

and

$$P_F\, a - 11.4\, \rho\, t^2\, c_m\, R\, u^2 > 0$$

Illustrative Examples

Assume a helicopter with the following data:

Gross weight, $G = 900$ kg

Rotor radius, $R = 6$ m

Tip speed, $u = 120$ m/s

Blade angle of attack, $\vartheta_o = 12^\circ$

Centrifugal force of each blade, $P_F = 1880$ kg

Number of blades, $z = 4$ (2 rotors of two blades each)

Blade chord, $t = 0.28$ m

Distance of hinge, $a = 0.2$ m

Inertia moment of blade, $J_F = 20$ kg/m/s²

Inertia moment of airplane about its lateral axis, $J = 150$ kg/m/s²

Distance of plane of rotor tip from airplane c.g., $s = 1.2$ m

Slope of lift lines for blade profile, $c_a' = 5.6$

Moment coefficient of blade profile referred to aerodynamic center, $c_m = 0$

Design factor, $\varphi = 1.0$

(Constant blades angle of attack during rise and sinking of blades)

These data correspond to a helicopter with the engine power and dimensions of an autogiro of type C 30. Equations (10), (11), (15), and (16) give in kg/m/sec:

$$S_{nv} = 3.45, \quad M_v = 6.80, \quad S_{n\dot{\alpha}} = 56.5, \quad M_{\dot{\alpha}} = 115$$

whence equation (17) reads

$$13800\,\nu^3 + (10550 + 750 + 520)\,\nu^2 + 15\,\nu + 6120 = 0 \qquad (21)$$

Insertion of the coefficients in (18) discloses the latter to be far from satisfied, as the positive term contains only about 1/470 of the negative term. The evaluation of equation (21) gives aside from a negative real root (rapidly decaying aperiodic motion) two conjugated complex roots $\nu = 0.16 \pm 0.60\ i$, which are identical with an amplified oscillation of $T = 2\pi/0.60 = 10.4$ s period and an amplification factor of $e^{0.16\ T}$. In other words, the amplitude increases during one oscillation $e^{0.16\ 10.4} = 5.2$ fold. In view of the long oscillation period, it should be possible to keep the self-induced oscillation of such a helicopter within narrow limits in spite of the great amplification.

In the factor of ν^2 in equation (17) the first term predominates, as a comparison with equation (21) discloses. Thus the stability condition, equation (18), can, with allowance for equation (19), be written in the form

$$M_{\dot{\alpha}}\ \frac{0.35 J_F P_F a z}{c_a' \omega^2 \rho\ R^5 t}\ \varphi \left(1 - \frac{11.4\ \rho t^2 c_m R u^2}{P_F\ a}\right) - J\ M_v > 0 \qquad (22)$$

Additional simplifications can be effected by assuming

$$0.044\ \frac{G}{u}\ s - \frac{z}{4}\ \ \rho\ t^2\ c_m\ R\ u = 0 \qquad (23)$$

because then the last two terms on the right-hand side of equation (11) cancel. For our example, equation (23) gives $c_m = 0.056$, i.e., a moment coefficient as is readily obtainable for a lift profile. With allowance for equations (11) and (16), equation (22) becomes

$$2.8 \left(\frac{J_F}{c_a' \omega\ \rho\ R^4\ \ t}\right)^2 \frac{\varphi\ P_F a z}{J\ i_0} \left(1 - \frac{11.4 \rho\ \ t^2 c_m R u^2}{P_F\ a}\right) > 1 \qquad (24)$$

from which the precautionary measures necessary to obtain stability can be read immediately. It involves, first of all, an increase in the mass of the rotor blades because then J_F, which enters the stability condition quadratically, as well as the centrifugal force P_F becomes higher. To get a starting point for the effect of the blade weight, we assume 3.5 times the value of the blade moment of inertia and 3 times the value of the centrifugal force in the foregoing example. With $J_F = 70\ kg/m/s^2$ and $P_F = 5600$ kg it affords $S_{nv} = 3.45$, $M_v = 12.0$, $S_{n\dot{\alpha}} = 200$, $M_{\dot{\alpha}} = 740$, and equation (17) reads

$$13800\ \nu^3 + (68000 + 1300 + 520)\nu^2 + 160\ \nu + 10800 = 0 \quad (25)$$

The stability equation (18) is still not complied with, although the positive term now amounts to a mere 1/13 of the negative term. The evaluation of equation (25) yields aside from a negative real root the conjugate complex roots $\nu = 0.02 \pm 0.40$ i, equivalent to an amplified oscillation with $T = 15.7$s period and $e^{0.02T}$ amplification factor. During one oscillation the amplitude increases $e^{0.02\ 15.7}$ = 1.3 times. In other words, the amplification is now substantially less and the oscillation period longer.

Naturally, equation (24) must be satisfied if stability is to prevail but it is not to be summarily concluded from the noncompliance with (24) that an increase on the left-hand side of equation (24) will, in every case, produce a lower amplification.

Take, for instance, the design factor $\varphi = 0$. In that case stability cannot be obtained according to (24). With otherwise identical data as in the first illustrative example we now have $S_{nv} = 0.33$, $M_v = 0.40$, $S_{n\dot{\alpha}} = 0$, $M_{\dot{\alpha}} = 0$, and equation (21) becomes

$$13800\ \nu^3 + (44 + 50)\ \nu^2 + 360 = 0 \quad (26)$$

It contains other than a negative real root the conjugate complex roots $\nu = 0.14 \pm 0.25$ i. The oscillation period is $T = 25$ s, substantially longer than in the first example, while the amplification factor $e^{0.14\ T}$ has become less. By decreasing M_v through raising the moment coefficient c_m in equation (11) the oscillation period can be further increased and the amplification factor lowered. Accordingly, there are several entirely different ways of rendering the spontaneous oscillations of a helicopter harmless.

Next, it can be proved that the original assumptions regarding the blade motion hold approximately true. To begin with the speed of the airplane in relation to the speed of rotation of the blades is very low. The oscillation periods of the airplane range between 10 and 25 seconds as against one third of a second for the blades. Hence the error following the assumption of a succession of stationary stages for the rotors cannot be very great.

Moreover, in the prediction of the forces and moments the premise stipulated constant position of the rotor-tip plane with respect to the fuselage. To check this premise we computed for the first example the ratio of speed amplitude v_0 to amplitude of airplane rotation about the transverse axis α_0 by writing the solution

$$v = v_0 e^{0.16\tau} \sin 0.60\,\tau$$

$$\alpha = \alpha_0 e^{0.16\tau} \sin (0.60\,\tau + \beta)$$

into equations (8) and (9). It yields $\beta = 70^\circ$ and $v_0/\alpha_0 = 15$. Suppose $v_0 = 1$; then $\alpha_0 = 3.8^\circ$. The amplitude of rotation is around $\dot{\alpha}_0 \approx 0.60\,\alpha_0$, which leaves the amplitude of the moments from the gyroscopic forces at $\dot{\alpha}\, M_{\dot{\alpha}} = \frac{0.60}{15}\, 115 = 4.6$ m/kg. This corresponds to an inclination of 0.14° of the plane of the rotor tip according to equation (5). The amplitude of the moments due to the horizontal speed is $v\, M_v = 6.8$ m/kg which, when disregarding the small thrust component in the plane of the rotor disk according to equation (6), is identical with a 0.21° inclination of rotor-tip plane according to equation (5). Thus we have a 3.8° airplane inclination against 0.14° and 0.21° inclination changes of the plane of rotor tip toward the fuselage axis, and this closely approaches the assumption for the prediction of the air loads and moments.

Lateral Stability

The study so far has dealt with longitudinal stability. Since only the air loads on the rotors have been considered the equations remain applicable if v and α are interpreted as lateral horizontal speed and as angle of rotation about the longitudinal axis. Then J must be included as inertia moment of the airplane about the longitudinal axis. With centrally arranged lifting propellers, the inertia moment about the longitudinal axis amounts to about one third that about the transverse axis, so that lateral stability is easier to obtain according to (24) than longitudinal stability. Placing two rotors side by side and the rotor centers farther apart, our assumption of small angles between flow velocity and rotor plane no longer holds true for rotation about the longitudinal axis. Suppose the distance of the rotor centers $2r$ is so great in relation to the distance s of the airplane c.g. from the plane of the rotor, that in rotation about the longitudinal axis the air strikes the rotors at an angle of around ±90°. For small coefficients of rotor advance λ in thrust direction, we have

$$\frac{k_s}{2\sigma} = \vartheta_o - \frac{3}{4}\lambda - \frac{3}{4}\sqrt{k_s}$$

The damping coefficient of the air-load damping follows at

$$M_{L\dot{\alpha}} = \frac{\partial M_L}{\partial \dot{\alpha}} = -\frac{2r^2}{u}\frac{\partial S}{\partial \lambda} = -r^2 \rho F u \frac{\partial k_s}{\partial \lambda} = \frac{r^2 \rho F u}{\frac{2}{3\sigma} + \frac{1}{2\sqrt{k_s}}} \quad (27)$$

For all other remaining forces, the validity of the previously derived relations is assumed, whence it then merely requires the replacement of $M_{\dot{\alpha}}$ in equation (17) by the sum of $M_{\dot{\alpha}}$ from equation (16) and $M_{L\dot{\alpha}}$ from equation (27).

Take, for instance, a helicopter with the following data:

Gross weight, $G = 900$ kg (9.8) ≈ 9000 newtons

Rotor radius, $R = 4$ m

Tip speed, $u = 120$ m/s

Blade setting, $\vartheta_o = 12^\circ$

Centrifugal force per blade, $P_F = 1250$ kg

Blades, $z = 6$ (two rotors side by side of 3 blades each)

Blade chord, $t = 0.25$ m

Hinge distance, $a = 0.2$ m

Blade moment of inertia, $J_F = 3.5$ kg/m/s^2

Airplane moment of inertia about longitudinal axis,
$J = 500$ kg/m/s^2

Distance of plane of rotor disk from c.g. of airplane,
$s = 1.2$ m

Lift gradient, $c_a' = 5.6$

Solidity ratio, $\sigma = 0.06$

Thrust coefficient, $k_s = 0.01$

Clearance between rotor centers, $2\ r = 9$ m

$c_m = 0$ and $\varphi = 1$, as before.

These data correspond to a helicopter of the power and approximate dimensions of the helicopter type FW 61. Equations (10), (11), (15), (16), and (27) give (in kg/m/s):

$S_{nv} = 3.45$, $M_v = 6.80$, $S_{n\dot{\alpha}} = 38$, $M_{\dot{\alpha}} = 77$, $M_{L\dot{\alpha}} = 950$

Then equation (17) reads

$$46000\ \nu^3 + (95000 + 750 + 1730)\nu^2 + (3560 - 260)\nu + 6120 = 0$$

Inserition of the coefficients in the inequation (18) discloses that the stability condition is exactly satisfied, since the positive term is 7 percent greater than the negative. There is therefore no difficulty in so designing such a helicopter as to assure dynamic stability in lateral motions.

Control

The question of control is closely allied with that of stability, since measures to influence the stability characteristics of an airplane usually also affect its control characteristics. On a helicopter with hinged blades the inertia moment of the airplane $\ddot{\alpha}\ J$ and the damping moment $\dot{\alpha}\ M_{\dot{\alpha}}$ must be overcome through the external control moment M_{St}, whereby $M_{\dot{\alpha}}$ is given through equation (16) or equation (27). Thus the angle of rotation α of the airplane under the control moment must be obtained under the same assumptions as before from equation

$$\ddot{\alpha}\ J + \dot{\alpha}\ M_{\dot{\alpha}} = M_{St} \qquad (28)$$

In the first example computed, we found $M_{\dot{\alpha}} = 115$; i.e., a control moment of 115 m/kg would induce a steady rotation at an angular velocity of $\dot{\alpha} = 1$ (57.3°/s). Hence the damping against rotations about the longitudinal and transverse axis in an autogiro with hinged blades is quite substantial even at zero flying speed. The solution of the differential equation (28) for abruptly applied control moment M_{St} from neutral position reads

$$\alpha = \frac{M_{St}}{M_{\dot{\alpha}}} \tau - \frac{J\ M_{St}}{M_{\dot{\alpha}}^2} \left(1 - e^{-\frac{M_{\dot{\alpha}}}{J} \tau}\right)$$

For the data of the first example $(M_{\dot{\alpha}} = 115,\ J = 150)$ a control moment $M_{St} = 10$ m/kg gives after 1 second an angle of rotation of the airplane of 1.5°, after 2 seconds, 4.9°. Raising the blade mass to about three times its value as in the latter example $(M_{\dot{\alpha}} = 740,\ J = 150)$ gives an angle of rotation of 0.8° after 1 second and of 1.6° after 2 seconds. Thus improving the stability by raising the mass of the rotating blades involves at the same time the danger of excessive inertia in the control action. By vanishing design factor φ and hence damping $M_{\dot{\alpha}}$, it again affords with $J = 150$ and $M_{St} = 10$ m/kg an angle of rotation of 1.9° after 1 second and of 7.6° after 2 seconds. Of course, for $\varphi = 0$, the amplification factor of the spontaneous airplane oscillations can, as was shown, be considerably reduced but not without danger of abnormal control sensitivity of the helicopter. Under what conditions the flight characteristics of a helicopter, taken as a whole, are most agreeable, is impossible to decide theoretically. It requires systematic flight tests under the different possible conditions.

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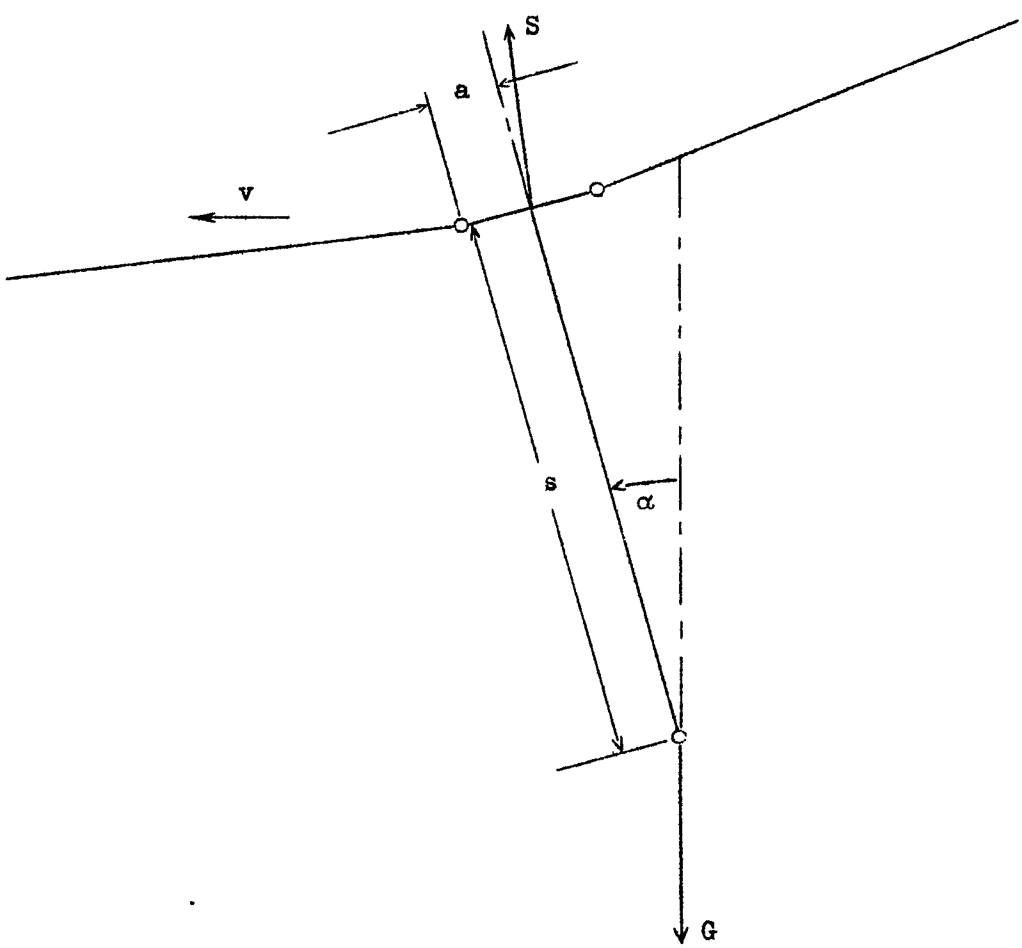

Figure 1.- Definition of positive directions for α and v.